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THE INFLUENCE OF BELIEFS IN THE PROCESS OF TEACHING-LEARNING MATHEMATICS

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Abstract
The importance of the affective domain in the general teaching-learning process, and in teaching mathematics in particular, has been analysed since the 1980s. Among the explanatory variables in this domain, beliefs play a determining role as they affect our self-perception, the perception of others, and the perception of learning and the situations that surround us, with the added particularity that this perception predisposes to action. In this process, the beliefs that each pole in the educational interaction hold about themselves, about the subject of mathematics, and about their role in the interaction with mathematics, will result in some sort of forecasting on what one can and cannot expect. Therefore, in this study we analysed the beliefs towards mathematics of students throughout their four years of secondary education and of the teachers of this subject in Northern Spain. We observe a great distance between students' and teachers' beliefs, and discuss the results obtained based on their educational implications.

Keywords: beliefs; teachers; students; mathematics; secondary education

Introduction

The subject of Mathematics is among those presenting the most difficulties for students during their academic life. The poor results obtained in international evaluations (MEC, 2013; OECD, 2010; PISA, 2012) and the low enrolment of students in this degree at university among others, are only some of the data that reflect this difficulty. The problems students have with mathematics

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are attributed, on many occasions, to the intrinsic conceptual complexity of this subject, but this explanation becomes narrow if the importance of other affective-emotional factors is not taken into account (Hidalgo, Maroto, & Palacios, 2005).

Even though the characteristics of mathematics –abstract, rigorous, exact, and demanding logical reasoning- cause it to be considered as "the science *par excellence*", students and teachers' beliefs about it are just as important as they play a role of implicit subjective knowledge that influence the perception of the difficulty of mathematics (Gómez- Chacón, 2000).

Among the components of implicit subjective knowledge not directly tied with the subject's difficulty, three perfectly differentiated groups can be distinguished: those related with the educational system (programs, methodology, etc.), those related to the social context (stereotypes, prejudices, etc.) and those related with the affective-emotional experience of the subject (attitudes, motivation, beliefs, mathematical self-concept, etc.) (Goldin, Rösken, & Törner, 2019; Gómez-Chacón, 1997; Hidalgo et al., 2005; McLeod, 1989, 1992).

This study falls into the category of those which attempt to explain the difficulties in mathematics according to the variables described in the third group mentioned, and more specifically, analyses the influence of beliefs in the process of teaching-learning of mathematics.

Theoretical framework

A belief is a feeling of certainty about the meaning of something. It is a personal affirmation, an individual thought (Southerland, Sinastra, & Mathews, 2001) that we hold as true. Beliefs, in many cases subconscious, affect our perception of both, ourselves and others, and that of things and situations around us. Beliefs are propositions held as true by an individual, not based on proof, but on personal judgement and evaluation (Luft, Roehring, Brooks, & Austin, 2003). Through our system of beliefs and values we give our world model coherence and meaning, a model we are so deeply attached to that we only very reluctantly change.

Beliefs stemming from personal experience accumulated throughout the teaching/learning life are some of the factors that determine the construction of mutual representations from which both, teachers and students perceive and value their respective actions (Coll et al., 1996). In this way, both poles elaborate some sort of prediction about what they can and cannot expect from each other,

to the point of directly or indirectly interfering in the conduct and relationships established. From this standpoint, Rosenthal and Jacobson (1968) put forward the hypothesis that the expectations developed by the teacher could have an influence on the student's learning outcomes. Along the same lines, Pajares (1992) maintains that teachers' beliefs influence their perceptions and judgements, which affect their behaviour in the classroom.

In the same way, some factors that determine the construction of mutual representations through which teachers and students perceive, value and orientate their respective actions within the subject of mathematics are the mental elaboration of the subject, the beliefs stemming from personal experience accumulated throughout their lives as students of this subject, and the importance that the context gives to mathematics - the family, school and friends contexts. These factors affect teaching and learning, affect the student role in this subject and even affect the social context in which mathematical education takes place.

In this sense, we can categorise beliefs according to their object, as (McLeod, 1992; Op´t Eynde, De Corte, & Verschaffel, 2002):

1. Beliefs about oneself regarding learning and resolving mathematical problems;
2. Beliefs about mathematics, their teaching and learning;
3. Beliefs about the social context in which mathematical education takes place.

In the first studies (McLeod, 1989, 1992), as later on (Estrada, 2002; Gil, Blanco, & Guerrero, 2005), the three groups of beliefs enumerated were already observed to influence not only the student learning process, but also how he or she would use this mathematical knowledge in the future. Moreover, beliefs predispose the individual to act in one way or another (Estrada, 2002). This is why beliefs should be considered as a cognitive reference that somehow influences behaviour.

Following the research carried out, and with regard to the first type of beliefs, *those referring to oneself as a learner*, attention has been focused mainly on the student self-trust for the following aspects: routine problem-solving; non-routine problem-solving; and learning fractions, proportions, algebra, geometry and calculus (Gómez-Chacón, 2002).

It is stated that students who perceived themselves as more capable are more willing to engage in tasks related with these learning fields, getting better qualifications as a result (Pajares, 2008; Rosario et al., 2009). Moreover, beliefs

in the capacity to reach the goals established seem to influence motivation and the effort put in learning, the quality of information processing -probably deeper-, and academic performance (Schunk & Ertmer, 2000).

Regarding the second type of beliefs, that is: *students' beliefs about mathematics as a discipline*, attention focuses mainly on two aspects: on the one hand, the subject's perceived importance and usefulness; on the other hand, success and participation in the mathematics class (Alomar, 2007; Chen & Zimmerman, 2007; Simpkins, Davis-Kean, & Eccles, 2006).

Although as a general rule mathematics is considered complex and difficult to understand, demanding effort in comprehending, and the use of superior cognitive strategies (De Corte, Verschaffel, & Op´t Eynde, 2000; Schoenfeld, 2002; Estrada, 2002; Vizcaíno & Manzano, 2017), at the same time, students express they think it is useful; a perception of usefulness that is positively related to performance. The concept of usefulness is the factor that most influences the level of interest in learning the subject (Gil et al., 2005).

Regarding the third type of beliefs, *those related with the context in which the education in mathematics takes place*, the focus is mainly on three aspects: first, beliefs about the teachers' task and their style of educational interaction; second, beliefs about the role and functioning of students in the classroom; and last, beliefs about socio-mathematical norms and practices in the classroom (Op´t Eynde et al., 2002).

Since Barberá and Gómez-Granell (1996), it has been pointed out that mathematical knowledge construction in the classroom, in the final stages proposed by many teachers presents, at least, the following characteristics: a high level of abstraction and generality, the elimination and disconnection from the perceptive and intuitive forms of representation of objects, situations and contexts; and essentially deductive nature that cannot be validated through phenomena or data from reality, but through an internal process of demonstration from definitions and fundamental axioms; a formal specific language fundamentally intended not to communicate phenomena or real situations, but to obtain internally consistent results; an essentially theoretical nature, impersonal and timeless, which suppresses intentions, emotions and affections.

However, in real life it is nonetheless true that mathematical knowledge allows to learn and solve non-mathematical systems (Bishop, 1999). In any event, analysis and construction of mathematical knowledge within the

classroom as the study of the procedure used to reach this construction, lead us to conclude, as Schoenfeld (1987) did, that having declarative and procedural knowledge about a problem does not guarantee the ability to use these at the right time.

In this sense, some strategies and procedures that could bring this science closer to students and teachers would be guiding students' learning toward understanding and solving problems (Pérez & Pozo, 1994); linking formal language with its referential meaning (Castro and Castro, 1997); using the students' previous and informal mathematical knowledge as a point of departure (Becker & Selter, 1996); progressively advancing towards increasingly high levels of abstraction and generalization (Verschaffel & de Corte, 1996), and explicitly teaching high-level mathematical strategies (Mayer, 1999) could be some of the contents and procedures used in bringing this science closer to the students and teachers.

The studies by Alonso Tapia (1997) point out that the degree of interest and effort students approach their work with, is consistent with the significance it has for them to learn it, with the possibilities they believe they have in overcoming the difficulties and with the cost, in terms of time and effort, they foresee it is going to take. Ultimately, this brings us to the different perspectives in education; different perspectives yield different descriptions of, for example: what is an organised classroom for a teacher; what is the role of a good teacher; on what basis does the teacher assert that learning is happening in the classroom; what central aspects need focused on to conclude that students are learning; or how does the teacher decide what to teach, how to represent it, how to question students on a particular subject (Trinidad, 2012).

Some perspectives envisage the teacher, as the holder of the knowledge, the informer, the conveyor of the content students need to learn. Some other perspectives envisage the teacher as a facilitator and driving force for the task that needs to be constructed along with the students, making sense of what has been learnt. In the second stance, teachers establish an affective relationship with students, paying attention to students' needs and opinions and take them into account. Thus, teachers value the importance of diversity, while at the same time underline the importance of emerging interrelations (Gómez-Chacón, 2000). From this dichotomy, some authors have pointed out that the beliefs of students about the context in which mathematical education takes place, such as the teacher's role and task, are of vital importance in the construction and

interpretation of the educational action; therefore, students may see it as something distant, impersonal or, on the contrary, as an educational action intertwined with affective relations (Bishop, 1988; Cobb & Yackel, 1998; Gómez-Chacón, 2000; Gómez-Chacón, Op't Eynde, & De Corte, 2006; Nunes, 1992; Op´t Eynde et al., 2002).

For all these reasons, grasping the beliefs of each pole in the educational interaction seems essential for better understanding the process of education and identifying the influence hat those beliefs may have both in the process of education and its results. Hence, this paper analyzes the beliefs of students and math teachers who interact with them. The beliefs of each other are compared, also analyzing whether the beliefs vary according to the academic level. Besides, the paper proposes ways of explaining the results obtained, which can help in the rethinking of the educational interaction.

Objectives

- To know the beliefs of the Secondary Education students and their Mathematics teachers.
- To compare the beliefs of teachers with those of students as a whole, as well as by academic level.
- To analyze issues that may be explanatory of the beliefs of both groups.

Method

Participants

The sample examined in this study consists of 245 secondary school students (56.4% men and 43.6% women) and the 6 teachers that make up the math department of the same school. The school the sample belongs to, is located in a town of approximately 30.000 inhabitants within the area of Greater Bilbao, in northern Spain. This town has an unemployment rate of 21.67%, half of the population have had only primary education or no formal education and the immigration rate is 6.5% (EUSTAT, 2017). We can say that the school belongs to a socially disadvantaged context, with a significant proportion of dysfunctional families, school performance difficulties and a significant number of foreign students.

Instruments

Beliefs towards learning mathematics, both from students and from teachers, were evaluated through the questionnaire EAM-AC-C created by Hidalgo et al. (2005). This instrument consists of 44 items, in which participants give their assessment on a Likert scale with five categories, ranging from 0 (totally disagree) to 4 (totally agree). Instrument reliability rates for this study were very good, with α=.824 for the total sample, α=.819 for students and α=.965 for the group of teachers.

Procedure

Data were collected on the beliefs of secondary education students (n=245) and the mathematics teachers of the school (n=6), with the aim of finding out whether there were any differences between the ideas of both groups around this subject and how to undertake it.

The board of the school participating in the study were informed and gave consent for the research. Later, parents and teachers of the students involved also gave their informed consent. Questionnaires were filled in anonymously, in two sessions during school hours in March 2016. Two members of the research team were responsible for their administration.

Results

First, all 44 items were analysed to confirm that they met the normality principles and to discern what type of tests would have to be used in the following steps. All items, except 35 (*mathematics is a discipline for which boys are better suited than girls*), met the criteria. In the case of item 35, an asymmetry index of 1.979 and a kurtosis index of 4.253 were observed, the latter above desirable values. Regarding the responses to this item, a considerable deviation from normality can be observed for a reason: 65.2% of the sample answered in total disagreement (0). It cannot be safely stated whether this is really the case or the response is due to social desirability; in any event, it was obvious that this item was not discriminating enough and did not meet the necessary requirements, so it was eliminated from subsequent analyses.

Table 1. Teachers' beliefs (n=6)

Belief	m	SD.
01. Learning mathematics is something that just a few people do	.67	.516
02. Most teachers our students have had have been good	2.00	1.414
03. One of the most important things to learn and pass mathematics is to study every day or almost every day	2.67	.816
04. The difficulties our students have or may have with mathematics are due to their own limitations	2.83	.753
05. The difficulties our students have or may have with mathematics are due to the difficulty of the subject	1.17	.408
06. The difficulties our students have or may have with mathematics are due to a lack of help	1.17	.408
07. The difficulties our students have or may have with mathematics are due to a lack of study	2.33	1.366
08. When students get good results in mathematics, this is due mainly to good luck	.17	.408
09. When students get good results in mathematics, this is due mainly to our help	1.00	.00
10. When students get good results in mathematics, this is due mainly to the help of people around them (family, classmates, friends…)	1.33	.816
11. When students get good results in mathematics, this is due mainly to their own effort and study	3.50	.548
12. When students get good results in mathematics, this is due mainly to their own capability	2.33	.816
13. When students get poor results in mathematics, this is due mainly to bad luck	.33	.516
14. When students get poor results in mathematics, this is due mainly to our lack of help	.67	.516
15. When students get poor results in mathematics, this is due mainly to the lack of help from people around them (family, classmates, friends…)	1.17	.408
16. When students get poor results in mathematics, this is due mainly to a lack of effort and study on their part	2.33	.516
17. When students get poor results in mathematics, this is due mainly to their lack of capability	1.00	.00
18. In mathematics they find it hard to decide what to do to get good marks	1.33	.516
19. Our students' results in mathematics depend on us to a large extent	1.33	1.033
20. Maths teachers in this school tend to be less accessible than teachers of other subjects	.40	.548
21. Our methods are more boring than those in other subjects	1.00	.632
22. Relationships between students and mathematics teachers in our school are more friendly than with teachers of other subjects	1.00	.000
23. People who like mathematics can be a little weird	.33	.516
24. Our students' friends don't care for mathematics	1.75	.957
25. I think some mathematics teachers do not trust in the performance of a large number of their students	1.40	.548
26. Our students get more distracted in mathematics than in other courses	1.50	.577
27. Dealing with a complicated problem, our students give up easily	2.50	1.000
28. Mastering mathematics will help them be successful in later studies	2.67	.516

Table 1. Teachers' beliefs (n=6) - *continued*

Belief	m	SD.
29. We take more interest in our students' progress and performance in mathematics than in other subjects	1.67	.816
30. Students understand that what we do in mathematics explains something about the real world but they don't appreciate it	1.83	.753
31. Being good students in mathematics makes them feel more valued and admired by their classmates	1.60	.548
32. I think there is a relationship between our students' antipathy towards mathematics and the teachers of this subject	1.00	.000
33. Our students' families are more concerned about the results and marks in mathematics than in other subjects	1.60	.894
34. When students have difficulties with mathematics they ask their family (parents, sibling…) for help	2.00	.894
35. Mathematics is a discipline for which boys are better suited than girls	.25	.500
36. Mathematics teachers are always ready to clarify doubts and difficulties in class for our students	3.17	1.169
37. When students don't understand something about mathematics, they learn it off by heart	1.83	.753
38. In Mathematics classes teachers value effort and acknowledge our students' daily work more than in other subjects	1.75	1.500
39. Learning mathematics requires a lot of discipline	2.00	.894
40. Studying mathematics, in general, is not entertaining	.83	.408
41. The skills used in class to solve mathematical problems have nothing to do with those used to solve problems in daily life	1.17	.408
42. Mathematics is a challenge to one's own capacities	2.50	1.049
43. Mathematics is very abstract and distant from reality	1.17	.408
44. I think the result they reach when trying to solve a problem is more important than the process followed	1.17	.983

Some of the results obtained in the test on teachers' beliefs about mathematics are interesting (*see* Table 1). An interesting result is the fact that they attribute the students' difficulties in this subject either to the students' own limitations or a lack of study, and not so much to the difficulty of the subject or the lack of support.

In the same way, they believe that when students obtain good qualifications in mathematics, it is basically due to personal effort and study, and they have the skills, while they obtain poor qualifications when they do not devote enough effort to studying.

Ultimately, in the teachers' opinion, the success or failure of students in mathematics is due to the students' own capacities and attitudes, and not so much to the support and methods used by teachers. Together with this, item 36 stands out for its high average (*Mathematics teachers are always ready to clarify doubts*

and difficulties to students in the classroom). In other words, the vision teachers have of themselves explains that they do not attribute students' poor results to the teachers' attitudes. But is this view shared by students?

Once the teachers' results were analysed, we compared them to the results obtained for students, with the aim of verifying whether there were outstanding differences between these two groups. Comparisons were carried out for the complete group of students (*see* Table 2), per school year (*see* Table 3 and following).

Table 2. Comparison between teachers' beliefs (n=6) and beliefs for all students (n=245)

	Average				
	Students	Teachers	Student' t	Sig.	Cohen's d
01. Learning mathematics is something that just a few people do*	1.39	.67	3.215	.016	.770
05. The difficulties I have or may have with mathematics are due to the difficulty of the subject	1.87	1.17	3.850	.006	.815
09. When I get good results in mathematics, this is due mainly to my teachers' help	2.07	1.00	15.606	.000	1.431
17. When I get poor results in mathematics, this is due mainly to my lack of capability	1.40	1.00	5.075	.000	.465
20. Maths teachers tend to be less accessible than teachers of other subjects	1.30	.40	2.026	.044	1.129
21. Mathematics teachers' methods are more boring than those in other subjects	1.72	1.00	2.680	.038	.783
22. My relationship with mathematics teachers is more friendly than with teachers of other subjects	1.38	1.00	5.567	.000	.514
32. I think there is a relationship between my antipathy towards mathematics and the teachers of mathematics	1.25	1.00	3.559	.000	.317
40. Studying mathematics, in general, is not entertaining	1.94	.83	6.001	.000	1.138
41. The skills I use in class to solve mathematical problems have nothing to do with those I use to solve problems in daily life	1.89	1.17	3.926	.005	.802

Note: *The statements shown correspond to the instrument completed by students. The instrument for teachers is similar, but statements have been adapted (*see* Table 1).

In the comparison between teachers' beliefs and those of all the students (Table 2), what stands out is that significant items revolve around two main ideas: first, the different vision they have in the methodological aspect, that is, students, compared to teachers, consider that teaching methods of mathematics are more boring than those of other disciplines (item 21), not entertaining (item 40) and

bear little relationship with daily life (item 41); the second idea in which both groups differ significantly is in the capacity to help, accessibility of the teachers in this matter and the sympathy/ lack of sympathy towards teachers. While students perceive these deficits, teachers do not perceive them in the same manner.

In the following tables (Tables 3-6), differences per academic year are analysed. As can be seen, the differences in items 9, 40, and 41 are significant in all years.

Table 3. Comparison between teachers' beliefs (n=6) and 1st-year students' beliefs (n=66)

	Average				
	Students	Teachers	Student' t	Sig.	Cohen's d
01. Learning mathematics is something that just a few people do	1.45	.67	2.964	.012	.780
04. The difficulties I have or may have with mathematics are due to my own limitations	1.58	2.83	-2.888	.005	-1.380
09. When I get good results in mathematics, this is due mainly to my teachers' help	2.48	1.00	12.597	.000	2.219
22. My relationship with mathematics teachers is more friendly than with teachers of other subjects	1.49	1.00	3.501	.001	.611
27. Dealing with a complicated problem, I often give up easily	1.21	2.50	-2.430	.018	-1.270
40. Studying mathematics, in general, is not entertaining	1.72	.83	3.993	.001	.998
41. The skills I use in class to solve mathematical problems have nothing to do with those I use to solve problems in daily life	1.68	1.17	2.299	.036	.561

Starting with a comparison between teachers and 1st-year students (*see* Table 3), the contrast in item 4 stands out: teachers attribute the difficulties in mathematics to a greater extent to the students' limitations. In the same way, teachers think students give in very easily, which would explain that teachers give less importance to the help they give.

Like 1st-year students, 2nd-year students (*see* Table 4) seek help in the methodological approach, and state that studying mathematics is not entertaining (item 40) to a greater extent than teachers do. Seeking help is maintained in 2nd year and clearly increases in 3rd year; then in 4th years it goes back to 1st and 2nd-year levels.

Table 4. Comparison between teachers' beliefs (n=6) and 2nd-year students' beliefs (n=82)

	Average				
	Students	Teachers	Student' t	Sig.	Cohen's d
09. When I get good results in mathematics, this is due mainly to my teachers' help	1.97	1.00	8.341	.000	1.329
17. When I get poor results in mathematics, this is due mainly to my lack of capability	1.44	1.00	3.138	.002	.488
20. Maths teachers tend to be less accessible than teachers of other subjects	1.26	.40	2.061	.042	1.136
22. My relationship with mathematics teachers is more friendly than with teachers of other subjects	1.42	1.00	3.832	.000	.612
32. I think there is a relationship between my antipathy towards mathematics and the teachers of mathematics	1.28	1.00	2.547	.013	.397
40. Studying mathematics, in general, is not entertaining	1.80	.83	4.468	.001	1.044
41. The skills I use in class to solve mathematical problems have nothing to do with those I use to solve problems in daily life	1.69	1.17	2.492	.028	.598

Regarding the vision students have about putting their capabilities into practice to solve practical problems (item 41), the distance between what students observe and what teachers observe is significant in all years, and yet again, especially in 3rd year. Also, students state they perceive a large distance between the capacities used to solve problems in class and those required in their daily lives. Next is the comparison between teachers' beliefs and 3rd-year students' beliefs (*see* Table 5):

Table 5. Comparison between teachers' beliefs (n=6) and 3rd-year students' beliefs (n=50)

	Average				
	Students	Teachers	Student' t	Sig.	Cohen's d
01. Learning mathematics is something that just a few people do	1.74	.67	3.829	.002	1.077
08. When I get good results in mathematics, this is due mainly to good luck	1.08	.17	3.852	.001	-.358
09. When I get good results in mathematics, this is due mainly to my teachers' help	1.74	1.00	4.157	.000	.831
10. When I get good results in mathematics, this is due mainly to the help of people around me	2.44	1.33	2.943	.037	1.062

Table 5. Comparison between teachers' beliefs (n=6) and 3rd-year students' beliefs (n=50) - *continued*

	Average				
	Students	Teachers	Student' t	Sig.	Cohen's d
14. When I get poor results in mathematics, this is due mainly to lack of help from the teachers	1.76	.67	3.717	.002	1.000
20. Maths teachers tend to be less accessible than teachers of other subjects	1.64	.40	2.556	.014	1.465
21. Mathematics teachers' methods are more boring than those in other subjects	2.36	1.00	4.530	.001	1.533
32. I think there is a relationship between my antipathy towards mathematics and the teachers of mathematics	1.46	1.00	2.571	.013	.514
36. Mathematics teachers are always ready to clarify doubts and difficulties in class	1.67	3.17	-2.446	.018	-1.146
40. Studying mathematics, in general, is not entertaining	2.51	.83	7.090	.000	1.910
41. The skills I use in class to solve mathematical problems have nothing to do with those I use to solve problems in daily life	2.30	1.17	4.764	.000	1.300

This is the year where the greatest statistically significant differences are found, and when the distance between students' beliefs and teachers' beliefs is the largest. For example, a large distance is observed between the need for teacher's help stated by students: students state needing help (items 10, 14, and 20), while teachers do not perceive such need. Actually, in item 36 teachers score significantly higher than students, where they state being ready to clarify doubts and difficulties, while students perceive, by far, the opposite.

Similarly, in addition to stating that studying mathematics is not entertaining (item 40), 3rd-year students also state that the methods used by teachers in this subject are more boring than in others (item 21). This item appears expressly in this year and not in other years.

Finally, Table 6 presents the comparison between teachers' beliefs and 4th-year students' beliefs.

One aspect deserves to be taken into consideration regarding items 5 and 6. Both items reflect that students comprehend that their difficulties tackling mathematics are due to its intrinsic difficulty and the lack of help, both aspects given little regard by teachers. If we relate the distance in

explaining why these difficulties occur with item 4, in which teachers state that difficulties in mathematics are due to the students' own limitations, then the incomprehension between both poles in the educational interaction is served.

Likewise, and similarly to what happens in second year, item 17 is significant, as it states that the students' poor results are the consequence of their poor capacity.

Table 6. Comparison between teachers' beliefs (n=6) and 4th-year students' beliefs (n=47)

	Average				
	Students	Teachers	Student' t	Sig.	Cohen's d
04. The difficulties I have or may have with mathematics are due to my own limitations	1.70	2.83	-2.728	.009	-1.296
05. The difficulties I have or may have with mathematics are due to the difficulty of the subject	2.27	1.17	4.588	.000	1.268
06. The difficulties I have or may have with mathematics are due to a lack of help	1.80	1.17	2.598	.017	.691
09. When I get good results in mathematics, this is due mainly to my teachers' help	2.02	1.00	8.172	.000	1.719
17. When I get poor results in mathematics, this is due mainly to my lack of capability	1.89	1.00	4.781	.000	1.009
22. My relationship with mathematics teachers is more friendly than with teachers of other subjects	1.43	1.00	2.817	.007	.580
40. Studying mathematics, in general, is not entertaining	1.89	.83	4.349	.000	1.165
41. The skills I use in class to solve mathematical problems have nothing to do with those I use to solve problems in daily life	2.11	1.17	3.916	.001	1.061

Conclusions

As regards to the first objective, like with other authors (Gómez-Chacón et al., 2006; Archambault, Janosz, & Chouinard, 2012; Lazarides & Watt, 2015; Levpušček & Zupančič, 2009), in this study we state the existing gap between the teachers' beliefs and the students' beliefs with regard to the confidence in solving routine and non-routine problems (Gómez Chacón, 2002), the beliefs about the perceived importance and usefulness, and participation in mathematics class (Alomar, 2007; Chen & Zimmerman, 2007; Simpkins et al., 2006; Steiner,

2007; Walker, 2007), and the beliefs about the teachers' tasks, the style of their educational interaction and the socio-mathematical practice in the classroom (Op´t Eynde et al., 2002).

In this sense, and as indicated in the second objective, if we compare the beliefs of students and teachers, it is observed that students consider that they don't have good teachers of mathematics, that teachers do not have confidence in them and that the teaching methodology used in the classroom is more boring than that of other courses.

Teachers, for their part, attribute the students' difficulties to the students' own limitations or lack of study and effort, to a large extent. Moreover, they believe they are good teachers, that the relationship with their students is more amiable than those of teachers of other disciplines, that they are more accessible, that studying mathematics is fun.

Once the distance existing between the beliefs of the two poles in the educational interaction are observed, it is important to highlight the need to work on the beliefs of teachers, since putting into practice methodological changes often just leads to the modification of an instrument. Unless we rethink the concept of teaching-learning along with these modifications, the beliefs and expectations on students' success will not change.

Just like with our study, Gil and Rico (2003) also point at the fact that most of the teachers do not feel responsible for the difficulties of mathematics. Along the same lines, Porlán and Rivero (1998), mention different types of hurdles that influence teachers' beliefs: those referring to the reproduction of internalised patterns, implicit theories and conceptions and the lack of confidence perceived by students; and those referring to stereotypes about school, curricular regulations and school time.

Another relevant issue for its significance and the implications derived from it, is the help provided by teachers. Students point at how important it is that their teachers show confidence in them, that they are ready to clarify doubts and that they are accessible. The question that arises is: does this constant need for teaching help portray a low confidence in their own personal abilities and capacities, and the belief in their own limitations in mathematics? If this were the case, this would certainly explain their under preparedness to carry out the tasks, and hence the poor results (Pajares, 2008; Rosario et al., 2009).

If we add, to the above-mentioned, that 1st-year students and teachers strongly sustain that students' difficulties in mathematics are due to their own

limitations; that 1st and 3rd-year students also consider that learning mathematics is something that just a few people do; that 3rd-year students attribute good qualifications to good luck; and that 2nd-year students attribute the lack of capacity to their poor results, then we should maybe question the level of development of the mathematical self-concept of the students participating in this study. Again, the need arises to insist on the influence that teachers' beliefs have in the development of their students. This, which is the case for any teaching-learning scenario, is especially relevant for teachers in disadvantaged socioeconomic and sociocultural contexts, in which students, sometimes, do not find the necessary positive attitudes to develop strategies for coping with difficulties, as highlighted by Archambault et al. (2012). Just like Núñez and González-Pienda (1994), also state that the subject's active involvement in the process of learning increases when they feel competent, that is, when they have confidence in their own capability and have high self-efficacy expectations.

This general tendency is maintained in each and every comparison between teachers and students in all the years analysed, with even more emphasis in third year. Possible explanations to this marked increase in third year could be found in the contents of this particular year, more abstract, and in the decrease of the perception of competence for mathematics (Hidalgo, Maroto, & Palacios, 2004, 2005).

This all points to underlining something that we point out in the third objective of the study, which refers to the explanation of the big distance between students and teachers, as well as intervention proposals. In this sense, it is necessary to underline the importance of considering the emotional dimension, and especially beliefs, in the process of teaching-learning, (Campos, 2003; Gómez-Chacón, 1999, 2000; Hart, 1989; Hidalgo, Maroto, & Palacios, 2000a, 2000b; Mandler, 1989; McLeod, 1988, 1992, 1994). We cannot emphasize enough how important it is that teachers do not limit themselves to simply transferring the knowledge of a discipline. Building self-confidence, a liking and enthusiasm for learning mathematics (Alsina, 2000) and understanding its usefulness are essential abilities to develop in their day-to-day teaching task (Alonso Tapia, 1997). Learning is not meaningless memorising, and contents are not ends by themselves, but just a means to interpret the world (Porlán et al., 2010).

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