INTERACTIVE PROBLEM SOLVING: ASSESSMENT AND RELATIONS TO COMBINATORIAL AND INDUCTIVE REASONING

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Abstract
This paper focuses on problem solving, especially interactive problem solving, and two types of reasoning: combinatorial reasoning and inductive reasoning. The purpose of the study is to examine the nature of interactive problem solving by (a) defining a two-dimensional measurement model of problem solving comprising two processes, knowledge acquisition and application; and (b) evaluating the relations among problem solving, combinatorial reasoning and inductive reasoning. The sample was drawn from 11-13 years old elementary school students in China (N=187). The data-gathering instruments were three tests measuring problem solving, combinatorial reasoning and inductive reasoning. All three tests were delivered to students via the eDia online assessment platform. Structural equation modeling was used to test for dimensionality and relationships. The internal consistencies of the assessment were good. Cronbach’s alpha for each test varied between .79 and .94. In the dimensionality testing, problem solving showed a significantly better model fit (p<.05) with the two-dimensional model consisting of knowledge acquisition and knowledge application. Moreover, the analysis indicated that problem solving acquired a strong predicting effect from combinatorial reasoning, and a moderate but significant effect from inductive reasoning. In addition, combinatorial reasoning showed a strong correlation with inductive reasoning. The results indicated that problem solving is a multi-dimensional cognitive process involving specific thinking skills. The findings contribute to defining the construction and components of problem solving, suggest that schools should focus on reasoning skills training to assist with students’ problem-solving ability development.

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Introduction

Interactive problem solving

In the past, education aimed at teaching students knowledge and skills, but nowadays education is “[...] about making sure that students develop a reliable compass and the navigation skills to find their own way through an increasingly uncertain, volatile and ambiguous world” (Schleicher, 2017, p. 3). Currently, society, technology and the environment are constantly changing, a situation that requires students to develop the ability to solve novel problems in their study or daily life. Problem solving is thus considered one of the most important 21st-century skills (Dede, 2010), and improving students’ problem-solving skills has become one of the main aims and challenges in contemporary education (Greiff, Holt, & Funke, 2013).

In consideration of the importance of problem solving, an increasing number of assessment projects have begun to include problem-solving skills as one of the assessment domains. The Programme for International Student Assessment (PISA) conducted by the Organisation for Economic Co-operation and Development (OECD) is one of the most influential and important international large-scale assessments at present. PISA “assesses the extent to which 15-year-old students have acquired key knowledge and skills that are essential for full participation in modern societies” (OECD, 2014a, p. 3), and problem solving has been included among its assessment subjects (OECD, 2014a, 2017).

The PISA problem-solving assessments, especially PISA 2012, are typical cases of interactive problem solving1 (OECD, 2014b). An interactive problem-solving process can be described as a series of non-routine actions which can help the problem solver to reach the goal state (Greiff, Holt, & Funke, 2013). It is characterized by interaction between the problem solver and the problem (Greiff, Holt, & Funke, 2013). Problem solvers are required to generate and integrate information about the problem through interaction

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1 Interactive problem solving has also been described as creative problem solving (see OECD, 2014b) and complex problem solving (see Funke, 2014; Molnár et al., 2017).
(knowledge acquisition) (Greiff & Funke, 2017), and to try and solve the problem according to the acquired information (knowledge application) (Greiff & Funke, 2017). From the description, it can be seen that an interactive problem-solving task is a complex process constituted by mental and practical activities. However, in the PISA assessments, there was no in-depth analysis of the internal construction of problem-solving skills, and there are few studies focusing on the influence of general cognitive skills on the interactive problem-solving process. The present study aims to gain further understanding of problem solving skills, and to explore the influence of inductive reasoning and combinatorial reasoning in the approach to problem solving.

**Measurement methods of interactive problem solving**

Nowadays, computer-based assessment is providing new possibilities and opportunities in educational research. It is replacing traditional paper-based testing in many areas, including interactive problem-solving assessment. The interaction between problems and problem solvers is the most important element in interactive problem-solving measurement, but it is a kind of interaction that can only be assessed by computer, not paper and pencil. According to previous research, there are three different computer-based interactive problem-solving measurement methods: (1) Microworlds (see Gardner & Berry, 1995); (2) formal frameworks (see Funke, 2001); and (3) minimal complex systems (Funke, 2014). This paper will focus on a specific form of interactive problem solving assessment using minimal complex systems, which is known as the MicroDYN approach (Funke, 2014).

MicroDYN is a mature problem-solving assessment approach that has been widely used in European countries (see Csapó & Molnár, 2017; Greiff, Krkovic, & Hautamäki, 2016; Greiff & Wüstenberg, 2014). It also has been applied in the PISA 2012 problem-solving assessment (OECD, 2014b). It is based on multiple complex systems within the linear structural equation (LSE) framework (Funke, 2001). In this approach, the relations between input variables and output variables can be described by linear structural equations. Each MicroDYN task contains up to three input variables (represented by A, B, and C), which are related to up to three output variables (represented by X, Y, and Z, see Figure 1; Greiff et al., 2013). The relations between the input and output values are various. Causal relations between input variables and output variables are called direct effects, while the effects originating and ending with
output variables are known as indirect effects (Greiff et al., 2013). Indirect effects can involve an output variable influencing another output variable (side effects, see Figure 1: Y to Z) or influence itself (eigendynamics, see Figure 1: X to X) (Greiff et al., 2013). The assessment contains two phases, knowledge acquisition and knowledge application. In the knowledge acquisition items, students had to interact with the system by changing the values of input variables, and observe the corresponding changes of output variables, so as to find out the relationships between input and output variables. In the knowledge application part, students had to solve the given problems by assigning appropriate values to the input variables, to make the output variables reach the required range (Molnár & Csapó, 2018).

**Figure 1. Structure of a typical MicroDYN task**

**Reasoning skills and problem solving**

Reasoning is a kind of general thinking skill (Pellegrino & Glaser, 1982), normally “understood as a generalized capability to acquire, apply and transfer knowledge” (Molnár et al., 2017, p. 127). It has significant influence in almost all higher-order cognitive skills and processes (Csapó, 1997), which include knowledge acquisition and knowledge application (Bisanz, Bisanz, & Korpan, 1994; Hamers, De Koning, & Sijtsma, 2000; Molnár et al., 2017) and the general problem-solving process (Molnár et al., 2013; Tomic, 1995). In this study, two major reasoning skills, combinatorial reasoning and inductive reasoning, have been chosen for analysis because their influence on problem solving has been discussed most frequently in previous studies.

According to Adey & Csapó’s (2012) definition, combinatorial reasoning is the process of creating complex constructs out of a set of given
elements that satisfy the conditions explicitly given or inferred from the situation. Information processing is a central constituent element in the problem-solving process (Frensch & Funke, 2014), and combinatorial reasoning skills are applied in some key activities of information processing such as strategy generation and application (Newell, 1993). Their functions include, but are not limited to, helping problem solvers to discover relationships between certain elements and concepts, and promoting their fluency of thinking when they are considering different strategies (Csapó, 1999). Moreover, even if problem solvers prefer a trial-and-error method in the interactive problem-solving environment, higher-level combinatorial reasoning skills can help them to summarize experience of failure and organize possible solutions.

As for inductive reasoning, it has been described as the cognitive process of acquiring general regularities by generalizing single and specific observations and experiences (Molnár et al., 2013). The discovery of regularities relies upon detecting similarities and/or dissimilarities concerning the attributes or relations to or between objects (Klauer, 1990). Inductive reasoning will be applied in information processing during the process of solving general problems (Mayer, 1998). Its influence on both knowledge acquisition and knowledge application has been analyzed and demonstrated in previous studies (Klauer, 1990; Hamers et al., 2000; Molnár et al., 2013). Such studies have indicated that inductive reasoning is one of the component skills for problem solving.

Objectives

The objective of this study is twofold. First, we examine the internal construction of problem solving and emphasize its dimensionality. Secondly, we examine the relationships among problem solving, combinatorial reasoning and inductive reasoning. More specifically, we intend to answer the following research questions:

1. Can problem solving be better understood in terms of the two dimensions, knowledge acquisition and knowledge application, than by a single dimension that subsumes the sub-processes?
2. What roles do combinatorial reasoning and inductive reasoning play in the cognitive process of problem solving?
Method

Participants
Some studies (e.g. Molnár et al., 2013) have suggested that the ages 11-13 are the most important time for students’ reasoning skills development. Therefore, the participants in this study were selected from this age group. A total of 187 Chinese primary school students participated in this study (85 boys and 102 girls; age M=11.93, SD=1.06).

Instruments

Online measurement tool for problem solving
The computerized instrument of problem solving assessment contained one introduction video, one trial task, and 18 items, all adapted from the MicroDYN approach. The items were translated into simplified Chinese, and students had three minutes to provide answers for each item. Figure 2 is the screenshot for the sample items, the left part comprising the sample knowledge acquisition item, the right part the sample knowledge application item.

Fig. 2. Sample items for the problem solving assessment

Online measurement tool for combinatorial reasoning
The combinatorial reasoning assessment instrument consisted of 12 items based on Pásztor and Csapó’s (2014) design. Students needed to use given elements to create combinations which satisfied the given requirement. According to the elements given in the tasks, the assessment can be divided into two sub-constructs, figural and verbal (Fig. 3). For the figural items (the left part of Fig. 3), students were required to select pictures to create different
combinations by the drag-and-drop operation. For the verbal items (the right part of Fig. 3), students were required to create combinations of the given letters and/or numbers, and type their answers into the input box.

**Online measurement tool for inductive reasoning**

The inductive reasoning assessment instrument was based on Pásztor, Molnár, Korom, Németh, & Csapó’s (2017) assessment. Students were required to discover the relationship between given elements and answer using the drag-and-drop method. The assessment contained 50 multiple-choice items, which consisted of four sub-constructs: figural series, figural analogy, number analogy and number series (Fig. 4).
Procedures

The whole assessment was carried out by the eDia (Electronic Diagnostic Assessment; Molnár, 2015) platform in the school’s ICT room in June and July, 2017. The feasibility and reliability of online assessment via the eDia platform in the Chinese context have been established by a pilot study (see Wu & Molnár, 2018). The assessment took one-and-a-half hours in total, divided into three sessions. Problem solving was the first test, followed by combinatorial reasoning and inductive reasoning. All the items were in simplified Chinese. Students’ scores were automatically calculated by the eDia platform.

Data analyses

Structural equation modeling (SEM; Bollen, 1989) was the main tool for data analysis in this study. It was used to test the construction of all three thinking skills assessed as well as the relationships between these skills. The model was computed by software Mplus (Muthén & Muthén, 2010). Maximum Likelihood (ML) estimation was used to create the model. Some fit indices such as the Tucker-Lewis Index (TLI), the comparative fit index (CFI) and the root mean square error of approximation (RMSEA), were computed by Mplus and serve to indicate the aptness of the model.

Results

Descriptive statistics

Table 1 shows the basic statistical information: the number of items, reliability coefficients (Cronbach’s alpha), assessment score mean and standard deviation for the subscales of problem solving, combinatorial reasoning and inductive reasoning. The reliability indices were satisfactory for every subscale, ranging from .79 to .94. The high internal consistencies confirmed that the assessment was reliable. The means for the problem solving and combinatorial reasoning tests ranged from 35% to 45%, which was a little lower than our assumed optimal value (40%-60%), but still ideal for analyzing. The mean values for the inductive reasoning subscales varied widely (38%-77%), which was caused by the different level of difficulty for each subscale. Students’ performance in inductive reasoning was close to our initial assumption, and also suitable for analyzing.
Table 1. Descriptive statistics for each assessed thinking skills and their subscales

<table>
<thead>
<tr>
<th></th>
<th>Number of Items</th>
<th>Cronbach’s alpha</th>
<th>Mean (%)</th>
<th>SD (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Problem Solving</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Knowledge acquisition</td>
<td>9</td>
<td>.87</td>
<td>45.34</td>
<td>32.30</td>
</tr>
<tr>
<td>Knowledge application</td>
<td>9</td>
<td>.79</td>
<td>35.06</td>
<td>26.08</td>
</tr>
<tr>
<td><strong>Combinatorial Reasoning</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Figural</td>
<td>6</td>
<td>.91</td>
<td>42.59</td>
<td>27.97</td>
</tr>
<tr>
<td>Verbal</td>
<td>6</td>
<td>.92</td>
<td>37.15</td>
<td>32.64</td>
</tr>
<tr>
<td><strong>Inductive Reasoning</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Figural series</td>
<td>11</td>
<td>.85</td>
<td>77.30</td>
<td>25.77</td>
</tr>
<tr>
<td>Figural analogy</td>
<td>15</td>
<td>.94</td>
<td>73.26</td>
<td>32.28</td>
</tr>
<tr>
<td>Number analogy</td>
<td>8</td>
<td>.82</td>
<td>51.40</td>
<td>30.85</td>
</tr>
<tr>
<td>Number series</td>
<td>8</td>
<td>.93</td>
<td>38.40</td>
<td>32.33</td>
</tr>
</tbody>
</table>

**Dimensionality of assessed thinking skills**

Based on the measurement instrument design, all three thinking skills assessed contain several subscales. Multi-dimensional models were built (1) to answer the first research question regarding problem solving’s dimensionality, and (2) to demonstrate combinatorial and inductive reasoning’s dimensionality as the preliminary work for SEM modeling. The goodness of fit indices for dimensionality testing are indicated in Tables 2, 3 and 4.

Table 2. Goodness of fit indices for testing the dimensionality of problem solving

<table>
<thead>
<tr>
<th>Model</th>
<th>Chi-square</th>
<th>df</th>
<th>p</th>
<th>CFI</th>
<th>TLI</th>
<th>RMSEA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-dimensional</td>
<td>103.57</td>
<td>65</td>
<td>.01</td>
<td>.99</td>
<td>.99</td>
<td>.06</td>
</tr>
<tr>
<td>2-dimensional</td>
<td>90.59</td>
<td>64</td>
<td>.05</td>
<td>.98</td>
<td>.98</td>
<td>.05</td>
</tr>
</tbody>
</table>

The two subscales for problem solving assessment were knowledge acquisition and knowledge application. Both one- and two-dimensional models showed good model fit. However, the Chi-square difference testing indicated a significant difference (Chi-square=12.98, df=1, p<.001), while two-dimensional model showed a better model fit. Therefore, problem solving should be described as a two-dimensional construction, consisting of knowledge acquisition and application, rather than in terms of a single dimension that subsumes the sub-processes.

Table 3. Goodness of fit indices for testing the dimensionality of combinatorial reasoning

<table>
<thead>
<tr>
<th>Model</th>
<th>Chi-square</th>
<th>df</th>
<th>p</th>
<th>CFI</th>
<th>TLI</th>
<th>RMSEA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-dimensional</td>
<td>232.82</td>
<td>33</td>
<td>.001</td>
<td>.76</td>
<td>.73</td>
<td>.17</td>
</tr>
<tr>
<td>2-dimensional</td>
<td>80.29</td>
<td>32</td>
<td>.001</td>
<td>.94</td>
<td>.93</td>
<td>.09</td>
</tr>
</tbody>
</table>
The two subscales for combinatorial reasoning were figural and verbal. The one-dimensional model showed a bad model fit, while the two-dimensional model fit can be considered acceptable. The difference testing indicated a significant difference (Chi-square=152.53, df=1, p<.001) between these two models. Thus, combinatorial reasoning is much more appropriately considered a two-dimensional model in SEM modeling.

Table 4. Goodness of fit indices for testing the dimensionality of inductive reasoning

<table>
<thead>
<tr>
<th>Model</th>
<th>Chi-square</th>
<th>df</th>
<th>p</th>
<th>CFI</th>
<th>TLI</th>
<th>RMSEA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-dimensional</td>
<td>204.67</td>
<td>64</td>
<td>.001</td>
<td>.954</td>
<td>.983</td>
<td>.11</td>
</tr>
<tr>
<td>2-dimensional(1)*</td>
<td>178.12</td>
<td>65</td>
<td>.001</td>
<td>.963</td>
<td>.986</td>
<td>.09</td>
</tr>
<tr>
<td>2-dimensional(2)*</td>
<td>167.09</td>
<td>66</td>
<td>.001</td>
<td>.967</td>
<td>.988</td>
<td>.09</td>
</tr>
<tr>
<td>4-dimensional</td>
<td>115.31</td>
<td>75</td>
<td>.001</td>
<td>.987</td>
<td>.996</td>
<td>.05</td>
</tr>
</tbody>
</table>

Note: *2-dimensional (1): figure-number; 2-dimensional (2): series-analogy

Inductive reasoning contains four subscales: figural series, figural analogy, number analogy and number series. Therefore, besides the one and four-dimensional models, it can also be built as a two-dimensional model, comprising figure-number and series-analogy. The one-dimensional model showed an unsatisfactory model fit. The two two-dimensional models’ model fits were acceptable, but still significantly worse than the four-dimensional model (p<.01). Therefore, inductive reasoning has been built as a four-dimensional construction in the following SEM model.

Figure 5. A structural model presents the relationships among problem solving, combinatorial reasoning and inductive reasoning
Relationship between assessed thinking skills

A SEM model has been built to examine the relationships among problem solving, combinatorial reasoning and inductive reasoning (Fig. 5). All three thinking skills are construed as latent variables composed of their sub-dimensions. The model fits were acceptable (Chi-Square:42.34, P<.001, CFI: .97, TLI: .96, RMSEA: .09, SRMR: .04). The model indicates that both knowledge acquisition and knowledge application were strong contributors to problem solving (β= .866-.883). The contribution from figural (β=.689) to combinatorial reasoning was significant but weaker than verbal (β=.857). As for inductive reasoning, it was strongly supported by figure analogy (β=.930) and number analogy (β=.881), while the contributions from other two dimensions were weaker but still high (β=.680-.794).

Both combinatorial reasoning and inductive reasoning showed a significant predicting effect for problem solving (p<.05), confirming these two reasoning skills’ importance in the problem-solving process. Moreover, the predicting effect of combinatorial reasoning (β=.611) was stronger than that of inductive reasoning (β=.241), indicating that the Chinese students solved problems by relying much more on their combinatorial reasoning skill. At the same time, combinatorial and inductive reasoning were highly correlated (r=.746, p<.01), proving that these two skills were impacting on each other in students’ cognitive development.

Conclusions

Our aims were to examine and analyze problem-solving skills’ internal construction and relationships with reasoning skills. In this study, we analyzed problem solving’s dimensionality and modeled the connections and influences among problem solving, combinatorial reasoning and inductive reasoning. Generally, the study proved that problem solving is not a skill with a simple structure but a complex cognitive progress consisting of knowledge acquisition and knowledge application, and involving specific reasoning skills.

Dimensionality of problem solving

To be more specific, in the assessment, participants were able to demonstrate their capacity for knowledge acquisition and application separately. The modeling results supported the hypothesis proposed by previous studies,
that problem solving could be formulated as a two-dimensional measurement model (e.g. Bühner et al., 2008; Wüstenberg et al., 2012). However, the two-dimensional model is still a simplified representation of the real-life problem solving progress. As Greiff et al. (2013) have illustrated, the complexities of naturalistic environments sometimes are much more extensive than the scenario simulated by the assessment instrument; and the model that is extracted from the collected data, inevitably, can only abstract and approximate the real situation. Nevertheless, the results of this study have built a foundation and provided possibilities for future research. In general, knowledge acquisition emphasizes understanding and representing the problem, while knowledge application emphasizes finding solutions (Greiff et al., 2013). Obviously, the processes of knowledge acquisition and knowledge application consist of complex mental and practical activities, which indicate possibilities for identifying lower-level dimensions within these two processes. Future research should focus on defining these sub-level component processes and further completing the construction of the problem-solving model.

Relationships between problem solving and reasoning skills

The study proved that combinatorial reasoning and inductive reasoning have significant predictive effects on the problem-solving process. The results indicated that both combinatorial reasoning and inductive reasoning were applied during the problem-solving process and affected the achievement of the process, although combinatorial reasoning’s influence was higher than inductive reasoning’s. Moreover, the results confirmed that combinatorial reasoning and inductive reasoning are strongly correlated, indicating that the development of problem-solving and other relevant reasoning skills are coordinated and not isolated. Currently, enhancing students’ ability to solve problems has become one of the main targets in school education, and this can be realized by explicit training (Molnár, 2011) or by improving teaching methods (Shayer & Adey, 2002). The findings of this study suggest that the problem-solving training programme should be accompanied by training in specific reasoning skills. Furthermore, certain school subjects have the capacity to promote reasoning skills development (e.g. mathematics education: Primi, Ferrão, & Almeida, 2010; Xin & Zhang, 2009; science education: Pásztor & Csapó, 2014; Kambeyo & Wu, 2018) - and thus further contribute to problem-solving ability development. The results suggest that schools can improve instruction methods in these subjects by paying more attention to
reasoning skills enhancement.

Limitations and future work

To conclude, this study contributes to the understanding of the nature of problem solving. However, that all the participants were from P. R. China may cause concern about the generalizability of the findings. Some studies (e.g. Csapó & Molnár, 2017) have pointed out that students from different nations could possibly have different levels of development in problem-solving performance, while the relationships between the components within problem solving skills could also vary. In order to overcome this issue, a further study has been designed. A cross-nation comparative study involving P. R. China, Hungary and Indonesia regarding the development levels and component skills of problem solving is in progress. The future study will discover such differences in the cognitive structures for problem solving as exist between students from different nations, and thus address the generalizability limitation of the present study.

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References


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